International Journal of Theoretical Physics, Vol. 41, No. 11, November 2002 (© 2002)

Asymmetric Brane-World Scenarios and Simulated Gravity

J. P. Uzan[1,2]

Received August 19, 2002

I review the main aspects of the simulated gravity that arises in brane-world models in which the reflection symmetry is broken. I recall its main aspects, show how a Newton-like force can be simulated on small scales, and discuss the post-Newtonian constraints as well as the cosmology of this model.

KEY WORDS: cosmology; gravitation; brane world.

1. INTRODUCTION

The suggestion by Randall and Sundrum (1999) that the effective four-dimensional gravity can be recovered on a four-dimensional brane embedded in a five-dimensional space-time (referred to as bulk) due to strong curvature effects has originated a lot of investigations of the, now known as, brane-world scenarios (see, e.g., Binétruy *et al.*, 2000).

Most of the works have focused on three-branes embedded in a five-dimensional space-time with reflection symmetry along the extradimension. This was originally motivated by the Hořava–Witten model (Horava and Witten, 1996) in which 11-dimensional M theory is compactified on the orbifold S_1/Z_2 and which can be reduced to an effective five-dimensional theory with three-branes located at the orbifold's fixed points. However, in general D-branes do not have to be supported at an orbifold fixed point.

Although originally physically motivated these two hypotheses (of dimensionality and symmetry) owe their popularity to their convenience. To derive the effective Einstein equations on the brane, one needs to evaluate the extrinsic curvature of the brane in terms of the matter content of the theory. In the case of a five-dimensional reflection symmetric space-time, it is completely determined by

[1] Laboratoire de Physique Théorique, CNRS-UMR 8627, Bât. 210, Université Paris XI, F–91405 Orsay Cedex, France; e-mail: uzan@th.u-psud.fr.
[2] Institut d'Astrophysique de Paris, 98 bis, Bd Arago, 75014 Paris, France.

the Israel junction conditions relating the discontinuity of the extrinsic curvature to the matter content of the brane, then allowing the four-dimensional geometry of the brane to be calculated by using the Gauss–Codacci relations (Shiromizu *et al.*, 2000). In more general cases where one has more transverse dimensions, the Gauss–Codacci equation relating the Ricci tensor of the brane to the geometry of the bulk can be generalized (see, e.g., Carter, in press) but there is no straightforward generalization of the Israel junction conditiond that apply only to the case of a hypersurface. Besides, when the reflection symmetry is relaxed, the Israel junction conditions alone are not sufficient to determine the extrinsic curvature and one needs to complement them with a dynamical equation for the brane motion.

Some specific models in which the reflection symmetry was broken in an ad hoc way by gluing two anti-de Sitter space-times (Davis *et al.*, 2001; Deruelle and Dolezel, 2000; Ida, 2000; Krauss, 1999; Perkins, 2001; Stoica *et al.*, 2000) with different cosmological constants have been studied. Recently, a natural mechanism for breaking the reflection symmetry by means of a gauge form field has been proposed (Battye and Carter, 2001; Carter and Uzan, 2001). Then, it was shown (Battye *et al.*, 2001) that, as long as the contributions of the bulk Weyl tensor are small enough (exactly in the same way as in the reflection symmetric case), the effective Einstein equations on the brane can be recovered without assuming any reflection symmetry and that they take the general form

$$\overline{G}_{\mu\nu} = -\Lambda_4 \overline{\gamma}_{\mu\nu} + 8\pi G_4 \overline{\mathcal{T}}_{\mu\nu} + \mathcal{O}(\overline{\mathcal{T}}^2), \tag{1}$$

where Greek indices referred to the background space-time, $\overline{\gamma}_{\mu\nu}$ is the first fundamental tensor, $\overline{G}_{\mu\nu}$ its Einstein tensor, and $\overline{\mathcal{T}}_{\mu\nu}$ the stress–energy tensor of the matter localized on the brane (see Section 2 for detailed definitions). The effective gravitational constant is given by

$$G_4 = \frac{3}{4\pi\overline{\mathcal{T}}_\infty}\left[(\pi^2 G_5 \overline{\mathcal{T}}_\infty)^2 - \left(\frac{\overline{f}}{4\overline{\mathcal{T}}_\infty}\right)^2\right], \tag{2}$$

where G_5 is the five-dimensional gravity constant, $\overline{f}$ is the normal component of the force applied on the brane and $\overline{\mathcal{T}}_\infty$ its bare tension. This work showed that the worldsheet geometry, and thus the apparent gravity on the brane, has three origins. There is a first part coming from the geometry of the bulk space-time, a second part arising from the discontinuity of the extrinsic curvature across the worldsheet and a third part coming from its mean value. In the reflection symmetry case, the third effect is absent (since the mean value of the extrinsic curvature vanishes).

The opposite limit in which the guenuine five-dimensional gravity is negligible, i.e. when $G_5 \to 0$, the preceeding approach ceases to be valid mainly because the contributions from the bulk Weyl tensor contain a coefficient G_5 in their denominator. Thus, they become dominant and one has to start from a fresh approach to deal with this apparently singular limit.

Here, we thus focus on the case where the effective four-dimensional gravity on the brane is recovered not by a confinement mechanism but by having a sufficiently small five-dimensional gravitational constant so that the genuine five-dimensional gravity is completely negligible.

In this approach, the four dimensional gravity is no more a fundamental force but an apparent effect due to the acceleration of a brane-world universe in a higher dimensional space-time. This quasi-gravity (Carter, 2000) or simulated gravity (Carter *et al.*, 2001) is thus a phenomenon that affects matter locally approximatively in the same way as true gravity, even though its origin is different and its detailed behavior may be rather different. To make a comparison, consider the case of the standard nonrelativistic Newtonian gravitational theory. In this context, the centrifugal force due to, e.g., the rotation of the Earth modifies locally the observable acceleration by a term to be added to the Newton attraction force. This contribution is indistinguishable from the Newton contribution in a crude experiment but differences will appear in more sophisticated experiments via, e.g., the Coriolis effect.

As an analogy to this simulated gravity force, consider the well-known experiment used to illustrate how relativistic gravity is a consequence of the space-time geometry and in which a mass, M say, is placed on a piece of fabric of tension $\mathcal{T}_\infty$. If we were performing this experiment far from all stars and galaxies, the mass will not bend the fabric and a test particle will remain at rest with respect to this mass: the external space is simply the Minkowski space and the situation is reflection symmetric with respect to the fabric so that the mass cannot bend it (see Fig. 1). If we were now performing this experiment in an accelerated lift, or simply on Earth, the acceleration of the lift or the Earth gravitational acceleration g will break the reflection symmetry and the mass will produce a dip in the level of the surrounding fabric. This will generate an effective two-dimensional effective potential, given by

$$V_{\rm eff}^{2D} = \frac{g^2}{2\pi\mathcal{T}_\infty} M \,\ln r, \tag{3}$$

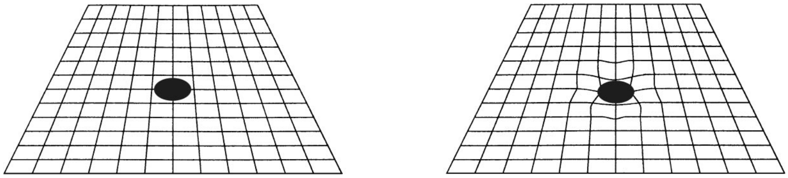

Fig. 1. If the true gravitation is negligible, a mass will not bend a reflection symmetric brane [left] whereas it induces a dip in the level of the surrounding brane when this symmetry is broken [right].

simulating a two-dimensional Newton-like force so that a test particle will orbit around the central mass in the same way as Earth orbit around the Sun. This example shows that the acceleration of the two-dimensional brane induces an effective two-dimensional Newton force in the brane even if the three-dimensional gravity is negligible. This two-dimensional effective gravity force is engendered by the geometry of the brane so that we will have a metric theory of gravity, hence satisfying the weak equivalence principal and the correct Newtonian limit. But, yet it can be noted that such a gravity law is described by only one degree of freedom (the displacement of the brane) contrary to standard gravity which has two degrees of freedom (the two polarization of the massless graviton).

In this paper, we first recall briefly in Section 2 the basics of brane dynamics, focusing on the equations that will be needed for our present purpose. We then investigate in Section 3 the small scales properties of this gravity force and will show that the standard Newton gravitational force can be recovered but that post-Newtonian effects put severe constraints on this force. In Section 4, we describe the cosmology with such a gravity and finish (Section 5) by discussing the viability and possible extention of this effect. Although it will suffer from a severe fine-tuning and the simplest model present here does not provide a viable alternative, the simulated gravity presented here is still of physical ineterest since it provides small but not necessary negligible corrections to the genuine effective gravitation (in the same sense as the centrigugal force provides corrections to the Newton attraction force on the Earth) as soon as the reflection symmetry is broken.

2. BASICS OF NONGRAVITATING BRANE DYNAMICS

We consider a three-brane (i.e. a four-dimensional worldsheet) embedded in a $(p+1)$-dimensional space-time. We follow the notations of (Carter, in press) to which the reader is referred for more details. Introducing the embedding functions $\bar{x}^\mu(\sigma^a)$ (with $\mu = 0..4$ and $a = 0..3$) defining the locus of the brane in terms of its internal coordinates σ^a, the five-dimensional metric $g_{\mu\nu}$ induces a metric γ_{ab} on the brane defined as

$$\gamma_{ab} \equiv g_{\mu\nu} \frac{\partial \bar{x}^\mu}{\partial \sigma^a} \frac{\partial \bar{x}^\nu}{\partial \sigma^b}, \tag{4}$$

which can then be mapped to the first fundamental tensor as

$$\gamma^{\mu\nu} \equiv \gamma^{ab} \frac{\partial \bar{x}^\mu}{\partial \sigma^a} \frac{\partial \bar{x}^\nu}{\partial \sigma^b}. \tag{5}$$

If we denote by n^μ the normal vector to the brane, the second fundamental tensor (also referred to as extrinsic curvature) is defined by

$$K_{\mu\nu} \equiv -\bar{\nabla}_\mu n_\nu, \tag{6}$$

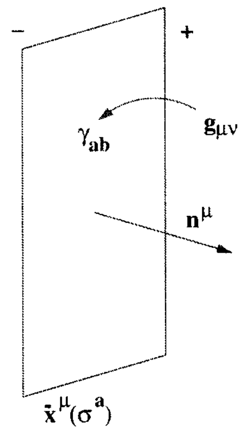

Fig. 2. The brane setup and the definition of the induced metric and embedding functions. + and − refer to the two sides of the brane and the unit normal vector n^μ points toward the region +.

where we have introduced the tangentially projected differentiation operator $\overline{\nabla}_\mu = \overline{\gamma}_\mu^\nu \nabla_\nu$, ∇_ν being the covariant derivative associated to $g_{\mu\nu}$. To finish, we use square and angle brackets to denote respectively the jump and the mean of any quantity

$$[A] = A^+ - A^- \qquad \langle A \rangle = \frac{1}{2}(A^+ + A^-), \tag{7}$$

where + and − refer to the two sides of the brane (see Fig. 2).

The Israel junction conditions relate the jump of the extrinsic curvature to the matter content of the brane. Since we assume that $G_5 = 0$, we deduce that

$$[K\overline{\gamma}_{\mu\nu} - K_{\mu\nu}] = 6\pi^2 G_5 \overline{\mathcal{T}}_{\mu\nu} = 0, \tag{8}$$

so that

$$[K_{\mu\nu}] = 0, \tag{9}$$

from which it follows that $\langle K_{\mu\nu} \rangle = K_{\mu\nu}$.

The dynamics of the brane is governed by the equation of motion (Carter, in press)

$$\overline{\mathcal{T}}^{\mu\nu} K_{\mu\nu} = \overline{f} = -\left[T^{\mu\nu}_{\text{bulk}}\right] n_\mu n_\nu, \tag{10}$$

which applies when self-gravity is negligible so that the bulk geometry is not affected by the brane and remains smooth. $\overline{f}$ is the orthogonal component of the external force density. Moreover the brane stress-energy tensor satisfies the conservation law

$$\overline{\gamma}_{\nu\rho} \overline{\nabla}_\mu \overline{\mathcal{T}}^{\mu\nu} = 0, \tag{11}$$

as long as the external force density has no tangential component. In the following, we shall consider only the case where this force arises from the minimal coupling to a gauge four-form for which $\overline{f}$ is simply a constant.

Eq. (10) is the generalization to any dimension of the Newton equation relating the acceleration of a body to the forces acting on that body. The extrinsic curvature is the generalization of the acceleration and the stress-energy tensor the one of the mass of the body. To have an intuitive understanding of its meaning, we can apply it to a standard two-dimensional bubble of radius R and of isotropic tension σ. The pressures inside and outside the bubble are respectively $P_{\rm in}$ and $P_{\rm out}$ so that $\overline{f} = -[T^{\mu\nu}_{\rm bulk}]n_\mu n_\nu = P_{\rm in} - P_{\rm out}$. The stress-energy tensor is just proportional to the first fundamental tensor, $\overline{\mathcal{T}}_{\mu\nu} = \sigma\overline{\gamma}^{\mu\nu}$ and the extrinsic curvature is given by the Gauss curvature $K_{\mu\nu} = \overline{\gamma}_{\mu\nu}/R$. It follows that Eq. (10) reduces to the standard relation $2\sigma/R = P_{\rm in} - P_{\rm out}$.

3. SMALL SCALES: NEWTON-LIKE FORCE AND POST-NEWTONIAN CONSTRAINTS

We first consider a perturbed configuration (see Fig. 3) of the brane setup presented in the previous section so that we can decompose the first and second fundamental tensors respectively as $\tilde{\overline{\gamma}}_{\mu\nu} = \overline{\gamma}_{\mu\nu} + \delta_{\rm L}\overline{\gamma}_{\mu\nu}$ and $\tilde{K}_{\mu\nu} = K_{\mu\nu} + \delta_{\rm L}K_{\mu\nu}$. Since the genuine gravitation is negligible, the geometry of the bulk remains unaffected and the only change in the metric tensor will be purely Lagrangian and given by

$$\delta_{\rm L}\overline{\gamma}_{\mu\nu} = \delta_{\rm L}g_{\mu\nu} = 2\nabla_{(\mu}\xi_{\nu)}, \tag{12}$$

where ξ^μ is the displacement vector. When working out perturbations, we always have to face a gauge freedom related to the freedom of the identification of the points of the perturbed manifold to the ones of the unperturbed manifold. We choose to fix the gauge by working in the orthogonal gauge defined by

$$\xi^\mu = \xi\, n^\mu. \tag{13}$$

It follows (see Carter *et al.*, 2001, for details) that

$$\delta_{\rm L}\overline{\gamma}_{\mu\nu} = -2\xi K_{\mu\nu}, \qquad \delta_{\rm L}K = \overline{\nabla}^\mu\overline{\nabla}_\mu\,\xi + \frac{\xi}{3}(\overline{R} + 4\,C^{\mu\nu}C_{\mu\nu} - 5\overline{\gamma}^{\mu\nu}\mathcal{S}_{\mu\nu}). \tag{14}$$

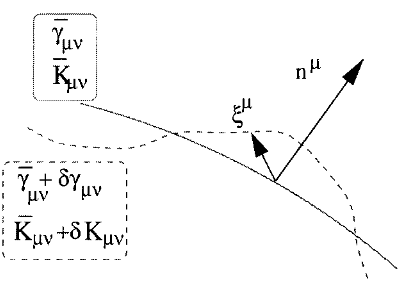

Fig. 3. Configuration of the perturbed brane (dash line) and of the unperturbed reference configuration (solid line). The definition of the displacement ξ^μ has a gauge freedom and the orthogonal gauge is defined by imposing $\xi^\mu = \xi n^\mu$.

where $C_{\mu\nu}$ is the tracefree part of the extrinsic curvature, $\mathcal{S}_{\mu\nu}$ the tracefree part of the bulk Ricci tensor and $\overline{R}$ the brane intrinsic Ricci scalar.

With these results, the perturbed version of the equation of motion (10) takes the form $\tilde{\overline{\mathcal{T}}}^{\mu\nu}\tilde{K}_{\mu\nu} = \overline{f}$ with a perturbed surface energy–momentum tensor of the form

$$\tilde{\overline{\mathcal{T}}}^{\mu\nu} \equiv -\mathcal{T}_\infty \tilde{\overline{\gamma}}^{\mu\nu} + \tilde{\overline{\tau}}^{\mu\nu}, \tag{15}$$

in which the constants $\mathcal{T}_\infty$ and $\overline{f}$ are just the same as in the reference configuration. By subtracting this dynamical equation from its unperturbed analogue (10) we are left with the dynamical source equation for the displacement ξ

$$\mathcal{T}_\infty \delta_{\rm L} K = K_{\mu\nu}(\tilde{\overline{\tau}}^{\mu\nu} - \overline{\tau}^{\mu\nu}) + \overline{\tau}^{\mu\nu}\delta_{\rm L} K_{\mu\nu}. \tag{16}$$

When $\overline{\tau}_{\mu\nu} = 0$ and $\tilde{\overline{\tau}}_{\mu\nu} = 0$, Eq. (16) can be interpreted, using (14), as the equation of evolution for a scalar field ξ nonminimally coupled to the Ricci scalar, $\overline{R}$, and with an effective mass given (of the kind discussed by Garriga and Vilenkin (1991)) by $m_\xi^2 \equiv -(4C^{\mu\nu}C_{\mu\nu} - 5\overline{\gamma}^{\mu\nu}\mathcal{S}_{\mu\nu})/3$.

Let us now consider the field in the neighborhood of a static spherical concentrated nonrelativistic source of integrated mass $\tilde{M}$, that is one whose energy-momentum contribution is given in terms of a preferred timelike unit reference vector $\overline{u}^\mu$ by

$$\tilde{\overline{\tau}}^{\mu\nu} - \overline{\tau}^{\mu\nu} \simeq \tilde{M}\delta(r)\overline{u}^\mu\overline{u}^\nu, \tag{17}$$

where r is the radial distance from the center. Considering a region small compared to the reference curvature scale so that we can work to first order both in $\overline{\tau}^{\mu\nu}/\mathcal{T}_\infty$ and in the displacement ξ, the second term of the r.h.s. of Eq. (16) is of order $\mathcal{O}(\xi\,\overline{\tau}^{\mu\nu}/\mathcal{T}_\infty)$ and thus negligible. On short lengthscales where we can keep only the highest order derivative terms, it leads to

$$\xi \simeq \left(\frac{-K_{00}}{(p-3)\Omega^{[p-2]}\mathcal{T}_\infty}\right)\frac{\tilde{M}}{r}. \tag{18}$$

It follows that the gravitational potential on the brane is given by

$$h_{00} \simeq 2G_{_4}\frac{\tilde{M}}{\overline{r}}, \tag{19}$$

if one identifies the unrationalized four-dimensional gravitational constant as

$$G_4 = \frac{K_{00}^2}{4\pi\mathcal{T}_\infty}. \tag{20}$$

It thus provides a plausible theory of Newtonian gravity but post-Newtonian effects (Will, 2001) (including light deflection, Shapiro effects . . .) imply that the local extrinsic curvature is very close to (see Carter *et al.*, 2001, for details)

$$K_{\mu\nu} = K_{00}\big(2\overline{u}_\mu\overline{u}_\nu + \overline{\gamma}_{\mu\nu}\big), \tag{21}$$

which indeed represents a very special configuration.

4. LARGE SCALES: A NONSTANDARD COSMOLOGY

The reference homogeneous configuration is solution of the Einstein vacuum equations, $\mathcal{S}_{\mu\nu} = 0$ and thus of the static form

$$ds^2 = r^2\, d\ell^2 + \frac{dr^2}{\mathcal{V}} - \mathcal{V}\, dt^2, \tag{22}$$

where $d\ell^2$ is the positive–definite space metric of a three-dimensional sphere, plane, or antisphere with constant curvature, k say, respectively positive, zero, or negative, and $\mathcal{V}$ is a function only of r. The location of the brane worldsheet will be given by an expression of the form $r = a(t)$. There will be an equivalent brane-based formulation

$$ds^2 = r^2\, d\ell^2 + d\zeta^2 - \nu^2\, d\tau^2, \tag{23}$$

in which the worldsheet locus is given simply by $\zeta = 0$, and in which τ represents proper time on this locus, so that $\nu = 1$ there. Using a dot to denote partial differentiation with respect to τ, and a dash for partial differentiation with respect to ζ, the rate of change of the scale factor a will satisfy the conditions

$$\dot{a}^2 = r'^2 - \mathcal{V} \quad \text{and} \quad \dot{a}^2 = \dot{r}^2/\nu^2. \tag{24}$$

Note that in the standard self-gravitating case the Birkhoff only applies in the cases where the matter in the bulk is either a cosmological constant (Bowcock *et al.*, 2000) or a gauge four-form (Carter and Uzan, 2001) but that in the case under consideration here where the true gravity is negligible it applies whatever the matter content of the bulk.

The function $\mathcal{V}(r)$ is given by $\mathcal{V}(r) = k - \Lambda r^2/6 - 2G_4\mathcal{M}/r^2$. We choose to set $k = 0$ on observational grounds favoring an almost spatially flat universe and have set $G_5 = 0$ in the spirit of this paper. It follows, from (24) and the conservation of the brane stress-energy tensor (11), that the expansion of the brane universe is governed by the Friedmann-like equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\overline{\Lambda}_4}{3} + \frac{1}{\mathcal{T}_\infty^2}\left[\frac{1}{(1+\varepsilon)^2}\left(\frac{\overline{f}}{4} + \frac{E}{a^4}\right)^2 - \left(\frac{\overline{f}}{4}\right)^2\right], \tag{25}$$

where $\varepsilon \equiv \overline{\rho}/\mathcal{T}_\infty$ and $\overline{\Lambda}_4/3 = \Lambda/6 + (\overline{f}/4\mathcal{T}_\infty)^2$, and where E is an integration constant that can be interpreted as the global energy of the system. Expanding in terms of ε we get

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\overline{\Lambda}_4}{3} + \frac{1}{\mathcal{T}_\infty^2}\left[\frac{\overline{f}}{2}\frac{E}{a^4} + \frac{E^2}{a^8} - 2\varepsilon\left(\frac{\overline{f}}{4} + \frac{E}{a^4}\right)^2\right] + \mathcal{O}(\varepsilon^2). \tag{26}$$

Usually (Binétruy *et al.*, 2000), E is bounded by the nucleosynthesis constraint. The idea of the present approach is that instead of being small, E is large enough to give a positive contribution that can overcome the negative contribution proportional to $-\varepsilon$. The cosmological scenario will thus have an era dominated by what we refer to as pseudoradiation, that will last until at least the present epoch unless it was superseded by a recent transition to an epoch of domination by a cosmological constant.

The requirements that the negative term in (26) does not dominate and to avoid premature domination by the cosmological constant implies that

$$4E > \bar{f}\varepsilon_{\rm r}^{0}\vartheta_{\rm eq}, \qquad \bar{f}\left(4E - \bar{f}\varepsilon_{\rm r}^{0}\vartheta_{\rm eq}\right) \geq \frac{8}{3}|\Lambda_4|\mathcal{T}_{\infty}^{2}, \tag{27}$$

where $\vartheta_{\rm eq} \equiv \varepsilon_{\rm m}^{0}/\varepsilon_{\rm r}^{0}$ is the ratio of the present energy densities in the matter and radiation. Nucleosynthesis implies that the four-dimensional Newton constant is identified with

$$G_{\rm N} = \frac{3\bar{f}}{16\pi\mathcal{T}_{\infty}^{3}}\left(\frac{E}{\varepsilon_{\rm r}^{0}} - \frac{\bar{f}}{4}\right), \tag{28}$$

and that

$$\bar{f}(4E - \bar{f}\varepsilon_{\rm r}^{0}) > 8E\left(E - \bar{f}\varepsilon_{\rm r}^{0}\right)\vartheta_{\rm N}^{4} \tag{29}$$

where $\vartheta_{\rm N} \sim 10^{10}$ (see Fig. 4).

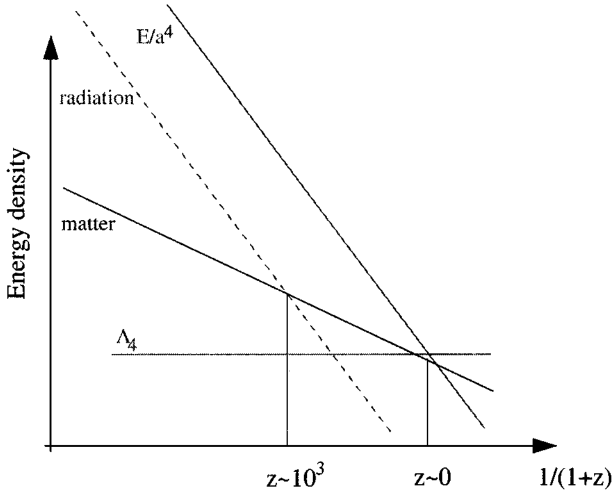

Fig. 4. Sketch of the cosmological scenario. The pseudoradiation component E/a^4, which scales light radiation, is supposed to dominate the matter content of the universe until recently where the cosmological constant starts to dominate.

5. VIABILITY

The simulated gravity model presented here provides a local gravitational interaction of the familiar Newtonian kind and can be contrived in such a way as to satisfy the most stringent cosmological constraint—-namely the one provided by nucleosynthesis. However it is still confronted with other serious problems.

1. It suffers from a double fine-tuning requirement arising from the fact that, on local scales, the gravitational constant depends a priori on time and that this time variation is observationnaly well constrained (Dickey *et al.*, 1994).
2. The predicted post-Newtonian effects and the current observational limits (Will, 2001) imply that the local value extrinsic curvature should turn out to be almost exactly of the form (21). Note that, at this stage of our investigation, these restrictions are not worse than those occuring e.g. for scalar–tensor theories of gravity (see Carter, 2000, for a comparison) before the mechanism of attraction (Damour and Nordtvedt, 1993) toward General Relativity was discovered.
3. In addition to the kinds of observational restrictions we have considered so far, it is to be noted that there will be others involving nonlinear effects and motion of the source as well as gravitational waves production.
4. Concerning the cosmology of our model, the expansion rate switches from a pseudoradiation to a cosmological constant dominated phase whereas the matter content switches from a radiation dominated era to a matter dominated era. This is a nonstandard cosmological scenario and in the case of the standard cosmology, it is known that density perturbations needed for subsequent galaxy formation cannot grow during a radiation dominated era. However it is not yet clear what may happen in a pseudoradiation era in which the strength of gravity on shorter scales is affected by the nature of fluctuations on longer scales.
5. The small scales and large scales determined four-dimensional effective gravitational constant are not equal. This discrepancy is not unreasonable in view that the brane configuration may have different curvature on small and large scales. We need a transition scale between these two behaviors that has to range somewhere above the galaxy scale. However, this scale is not directly determined by the model and might involve a third fine-tuning. A potentially relevant test that can be thought of is weak gravitational lensing on cosmological scales (Uzan and Bernardeau, 2001) which can provide a probe of the Newtonian law of gravitation (and more particularly of the Poisson equation) up to some hundreds of megaparsecs when data are available. This may be able to provide constraints on this transition scale.

The present model of simulated gravity, which originates from the breaking of the reflection symmetry in brane-world scenarios, reproduces a Newton-like gravity on small scales. The simplest version presented here suffers from many other problems arising from the post-Newtonian and cosmological properties of this gravity. We emphasize that in this model the gravity is tensorial and given by a space-time description but that it has only one degree of freedom. It extends previous investigations (Kehagias and Kiritsis, 1999) that were restricted to the cosmological context by investigating the small scale behavior and by allowing realistic matter on the brane. It is not a scalar gravity and the universality of free fall is not violated on the brane. The model can be improved in many ways by allowing matter such as a scalar field in the bulk, by changing the number of extradimensions or the bulk space-time structure. Besides, this gravity component is always present in models where the reflection symmetry is broken and in any moving brane model and, even if it has to be small, it may not be negligible.

ACKNOWLEDGMENTS

I warmly thank Brandon Carter, Richard Battye, and Andy Mennim with whom the work presented here was carried out.

REFERENCES

Battye, R. A. and Carter, B. (2001). *Physics Letters B* **509**, 331.
Battye, R., Carter, B., Mennim, A., and Uzan, J.-P. (2001). *Physical Review D: Particles and Fields* **64**, 124007. *Preprint* hep-th/0105091.
Binétruy, P., Deffayet, C., and Langlois, D. (2000). *Nuclear Physics B* **565**, 269.
Bowcock, P., Charmousis, C., and Gregory, R. (2000). *Classical Quantum Gravity* **17**, 474.
Carter, B. (2000). In *Proceedings of the Worshop on Analog Models for General Relativity*, Rio, Brazil, October 2000. Ed. Hatt visser *Preprint* hep-th/0106037.
Carter, B. (2001). International Journal of Theoretical Physics. **40,** 2099. *Preprint* gr-qc/0012036.
Carter, B. and Uzan, J.-P. (2001). *Nuclear Physics B* **606**, 45.
Carter, B., Uzan, J.-P., Battye, R., and Mennim, A. (2001). *Classical Quantum Gravity* **18**, 4871.
Damour, T. and Nordtvedt, K. (1993). *Physical Review D: Particles and Fields* **48**, 3436.
Davis, A.-C., Davis, S., Perkins, W. B., and Vernon, I. R. (2001). *Physics Letters B* **504**, 254.
Deruelle, N. and Dolezel, T. (2000). *Physical Review D: Particles and Fields* **62**, 103502.
Dickey, J. O., Bender, P. L., Faller, J. E., Newhall, X. X., Ricklefs, R. L., Ries, J. G., Shelus, P. J., Viellet, C., Whipple, A. L. Wiant, J. R., Williams, J. G., and Yoder, C. F. (1994). *Science* **265**, 482.
Garriga, J. and Vilenkin, A. (1991). *Physical Review D: Particles and Fields* **44**, 1007.
Horava, P. and Witten, E. (1996). *Nuclear Physics B* **460**, 500.
Ida, D.(2000). *The Journal of High Energy Physics* **0009**, 014.
Kehagias, A. and Kiritsis, E. (1999). *The Journal of High Energy Physics* **9911**, 022.
Krauss, P. (1999). *The Journal of High Energy Physics* **9912**, 011.
Perkins, W. B. (2001). *Physics Letters B* **504**, 28.
Randall, L. and Sundrum, R. (1999). *Physical Review Letters* **83**, 46090.
Shiromizu, T., Maeda, K., and Sasaki, M. (2000). *Physical Review D*: *Particles and Fields* **62**, 024012.
Stoica, H., Tye, H., and Wasserman, I. (2000). *Physics Letters B* **482**, 205.
Uzan, J.-P. and Bernardeau, F. (2001). *Physical Review D: Particles and Fields* **64**, 083004.
Will, C. (2001). *Living Reviews in Relativity* **4**, 2001–2004.